Buoyancy-drag mix model obtained by multifluid interpenetration equations

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In this paper, a buoyancy drag equation for describing the motion of the edges of a mixing zone driven by Rayleigh-Taylor or Richtmyer-Meshkov instabilities is derived from the multifluid interpenetration mix model equations of Scannapieco and Cheng [Phys. Letters A **299**, 49 (2002)]. This derivation provides a physics foundation for a large class of phenomenological buoyancy-drag mix models and also establishes a physical connection between the microscopic collision frequency and the macroscopic fluid drag coefficient. The predicted values for model parameter $\alpha^{ss'}$ in the multifluid interpenetration mix model, from the Rocket-Rig experiments, is in the range of 0.043–0.125 depending upon the Atwood number. The results are also in good agreement with inertial confinement fusion capsule implosions.

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I. INTRODUCTION

The studies of hydrodynamic instabilities of fluid interfaces have been carried out extensively in experiments [2–5], theories [6–17], and numerical simulations [6,8,19,20] over many decades, due to their strong effects on the implosion of inertial confinement fusion (ICF) targets [21,22], astrophysics, and industrial applications. A large fraction of the scientific work concerns the prediction and understanding of the growth rate of the instability [10–17], while a smaller but growing fraction involves the development of stochastic models to describe the structure and evolution of the entire mixing layer [1,8,16,18]. Depending on the physical assumptions underlying each mixing model, there is either an independent prediction of the instability growth rate [1,8] or one or two extra degrees of freedom that permit a separate model for the growth rate [23]. Particularly, in recent years, various models for fluid mixing driven by instabilities have been proposed, from multifluid turbulent mix models describing the microstructure and evolution of the mixing region [1,8,18,16] to simple buoyancy drag models predicting the growth rate of instabilities and large structures in the mixing layer [10–17]. The former are usually derived from the collisional Boltzmann equation, the Euler or Navier-Stokes equations and contain collision terms or interfacial source terms accounting for the interactions between species or forces exerted by one material on the other. The later are phenomenological and closed by drag coefficients describing the coherent structures such as bubbles and spikes in the mixing zone. In the past, these two classes of the models have been treated as independent, for example, the buoyancy drag equation has been used as an independent boundary condition in some multiphase flow models [13]. However, the derivation of the buoyancy-drag equation from physics principles remains unclear. The purpose of this work is to derive the buoyancy-drag equation from the multifluid interpenetration mix model equations and to provide a physical connection between the microscopic collision frequency which emerges in the multifluid averaged equations and the macroscopic fluid drag coefficient, which appears in the buoyancy-drag equation. In this paper, we will show that a multiphase flow model can, indeed, correctly reduce to a buoyancy-drag equation. First, we will present the multifluid mix model equations in Sec. II and then the detailed mathematical derivation from multifluid equations to the buoyancy-drag equation in Sec. III. In Sec. IV, we establish a profound relationship between the microscopic collision frequency in the multifluid interpenetration mix model and the macroscopic drag coefficeint in the buoyancy drag equation. Finally, a summary and further discussion are presented in Sec. V.

II. THE MULTIFLUID MIX MODEL

Recently, Scannapieco and Cheng [1] have proposed a multifluid interpenetration mix model, in which the set of multifluid moment equations (with or without external field) was derived rigorously from the collisional Boltzmann equation in a self-consistent manner. The model equations are mathematically complete and physically consistent with only one free parameter $\alpha^{ss'}$. In particular, the conservation equations for mass and momentum of each species s are written as follows:

$$\frac{\partial}{\partial t}\rho^s + \frac{\partial}{\partial x_j}(\rho^s v_j^*) + \frac{\partial}{\partial x_j}(\rho^s\langle U_j^s\rangle) = S_{\text{coll}}^s, \tag{1}$$

$$\frac{\partial}{\partial t}(\rho^s\langle U_j^s\rangle) + \frac{\partial}{\partial x_i}(v_j^*\rho^s\langle U_j^s\rangle) + \frac{\partial}{\partial x_i}\left(P_{ij}^s + \rho^s\langle U_i^s\rangle\frac{\partial}{\partial x_j}v_j^*\langle U_j^s\rangle\right) + \rho^s\left(\frac{\partial}{\partial t}v_j^* + v_i^*\frac{\partial}{\partial x_i}v_j^* + \langle U_i^s\rangle v_j^* - \langle g_j^s\rangle\right) = (A_j^s)_{\text{coll}}, \tag{2}$$

where ρ^s, v_j^*, and $\langle U_j^s\rangle$, respectively, express the species mean-mass density, mean-mass weighted bulk flow velocity, and peculiar velocity. $P_{ij}^s \equiv \rho^s\langle(U_i^s-\langle U_i^s\rangle)(U_j^s-\langle U_j^s\rangle)\rangle \equiv P^s\delta_{ij} + \Pi_{ij}^s$ represents the species thermal stress tensor, here P^s is the scalar pressure of species s, δ_{ij} is the Kronecker delta, and Π_{ij}^s represents the viscous stress tensor of species s. g_j is an external acceleration. A constant g_j corresponds to the Rayleigh-Taylor (RT) mixing and a shock or an impulsive

$g_j=\delta(t)$ describes Richtmyer-Meshkov (RM) mixing. The collision terms $S^s_{\rm coll}$ and $(A^s_j)_{\rm coll}$ in Eqs. (1) and (2) are expressed as

$$S^s_{\rm coll}=\sum_{s'}\delta M^s N^s \nu^{ss'}_{\rm eff}, \qquad (3)$$

$$(A^s_j)_{\rm coll}=\sum_{s'}[\rho^s(\langle U^{s'}_j\rangle-\langle U^s_j\rangle)+N^s\delta M^s\langle U^s_j\rangle]\nu^{ss'}_{\rm eff}, \qquad (4)$$

where N^s and M^s are the number density and mass of particle s, respectively, and δM^s is the change of M^s during the collision. $\nu^{ss'}_{\rm eff}$ is an effective collision frequency

$$\nu^{ss'}_{\rm eff}=\frac{4\pi M^{s'}}{M^s+M^{s'}}\frac{|\vec{U}^s-\vec{U}^{s'}|}{\lambda_{ss'}}\sim\frac{\rho^{s'}}{\rho^s+\rho^{s'}}\frac{|\vec{U}^s-\vec{U}^{s'}|}{\lambda^{ss'}}, \qquad (5)$$

where

$$\lambda^{ss'}=\lambda_c+\alpha^{ss'}L \qquad (6)$$

is the mean free path of particle s in which λ_c is the normal plasma collision mean free path, and the second term is the mean free path enhancement due to fluid instabilities, which is proportional to the total width of the mixing layer L that satisfies $dL/dt\equiv\partial L/\partial t+v^*_i\,\partial L/\partial x_i=|\langle U^s_j\rangle-\langle U^{s'}_j\rangle|$ or $L\equiv\int|\langle U^s_j\rangle-\langle U^{s'}_j\rangle|dt$. $\alpha^{ss'}$ is a model parameter to be discussed in Sec. IV.

III. DERIVATION OF THE BUOYANCY-DRAG MODEL

Expanding the terms in Eq. (2), introducing the total species velocity $\langle v^s_j\rangle\equiv v^*_j+\langle U^s_j\rangle$, and using Eq. (1), we rewrite Eq. (2) as

$$\rho^s\left(\frac{\partial}{\partial t}\langle v^s_j\rangle+\langle v^s_i\rangle\frac{\partial}{\partial x_i}\langle v^s_j\rangle\right)=\rho^s g^s_j-\frac{\partial}{\partial x_i}P^s_{ij}+(A^s_j)_{\rm coll}-\langle U^s_j\rangle S^s_{\rm coll}. \qquad (7)$$

Clearly, this is just the so-called Navier-Stokes equation if the collision terms vanish. In the case of zero viscosity ($\Pi^s_{ij}=0$) and zero mass transfer ($\delta M^s=0$, $S^s_{\rm coll}=0$) during the collision, let $d^s/dt\equiv(\partial/\partial t)+\langle v^s_i\rangle\partial/\partial x_i$, then Eq. (7) becomes

$$\rho^s\frac{d^s\langle v^s_j\rangle}{dt}=\rho^s g^s_j-\frac{\partial}{\partial x_j}P^s+(A^s_j)_{\rm coll}. \qquad (8)$$

This equation describes the dynamical evolution of the momentum of species s. Similarly, the dynamical equation of the momentum for species s' is expressed as

$$\rho^{s'}\frac{d^{s'}\langle v^{s'}_j\rangle}{dt}=\rho^{s'}g^{s'}_j-\frac{\partial}{\partial x_j}P^{s'}+(A^{s'}_j)_{\rm coll}. \qquad (9)$$

We apply Eqs. (8) and (9) to an interface between two fluids. Subtracting the s' from the s momentum equation and letting $g^s_j=g^{s'}_j=g_j$ gives

$$\rho^s\frac{d^s\langle v^s_j\rangle}{dt}-\rho^{s'}\frac{d^{s'}\langle v^{s'}_j\rangle}{dt}=(\rho^s-\rho^{s'})g_j-\frac{\partial}{\partial x_j}(P^s-P^{s'})+(A^s_j)_{\rm coll}-(A^{s'}_j)_{\rm coll}. \qquad (10)$$

At the edges of the mixing zone, the pressure of the vanishing fluid s per unit length in the direction of motion is equivalent to a force acting on the emerging fluid s' which moves at the same acceleration as the vanishing fluid [24,25], that is,

$$\frac{\partial}{\partial x_j}P^s=\kappa^s\rho^{s'}\frac{d^s\langle v^s_j\rangle}{dt},\quad \text{and}\quad \frac{\partial}{\partial x_j}P^{s'}=\kappa^{s'}\rho^s\frac{d^{s'}\langle v^{s'}_j\rangle}{dt}, \qquad (11)$$

where κ^s (or $\kappa^{s'}$) is a proportional coefficient which is also called the added mass coefficient, it equals 1 for cylindrical bubbles (spikes) and 1/2 for spherical bubbles (spikes) [25].

Incorporating Eq. (11) into Eq. (10), we have

$$(\rho^s+\kappa^s\rho^{s'})\frac{d^s\langle v^s_j\rangle}{dt}-(\rho^{s'}+\kappa^{s'}\rho^s)\frac{d^{s'}\langle v^{s'}_j\rangle}{dt}=(\rho^s-\rho^{s'})g_j+(A^s_j)_{\rm coll}-(A^{s'}_j)_{\rm coll}. \qquad (12)$$

The collisional terms in Eq. (12) are given by Eq. (4). For two fluids, these terms have expressions

$$(A^s_j)_{\rm coll}=\rho^s(\langle U^{s'}_j\rangle-\langle U^s_j\rangle)\nu^{ss'}_{\rm eff}, \qquad (13)$$

and

$$(A^{s'}_j)_{\rm coll}=\rho^{s'}(\langle U^s_j\rangle-\langle U^{s'}_j\rangle)\nu^{s's}_{\rm eff}. \qquad (14)$$

Using identities $\langle U^s_j\rangle-\langle U^{s'}_j\rangle\equiv\langle v^s_j\rangle-\langle v^{s'}_j\rangle$, we further have

$$(A^s_j)_{\rm coll}-(A^{s'}_j)_{\rm coll}=-(\rho^s\nu^{ss'}_{\rm eff}+\rho^{s'}\nu^{s's}_{\rm eff})(\langle v^s_j\rangle-\langle v^{s'}_j\rangle). \qquad (15)$$

If the two fluids have similar mass densities, then $\nu^{ss'}_{\rm eff}\approx\nu^{s's}_{\rm eff}$. By Eq. (5), Eq. (15) reduces to

$$(A^s_j)_{\rm coll}-(A^{s'}_j)_{\rm coll}=-(-1)^s\rho^{s'}\frac{(\langle v^s_j\rangle-\langle v^{s'}_j\rangle)^2}{\lambda^{ss'}}, \qquad (16)$$

here we have written $(\langle v^s_j\rangle-\langle v^{s'}_j\rangle)\equiv(-1)^s|\langle v^s_j\rangle-\langle v^{s'}_j\rangle|$ and denoted $s=1=b$ for the bubbles (the light fluid) and $s=2$ for the spikes (the heavy fluid). We also have assumed that $(-1)^s v^s(t)>0$ for all t.

Similarly, if the density difference of the two fluids is large ($\rho^s\gg\rho^{s'}$), then $\nu^{ss'}_{\rm eff}\ll\nu^{s's}_{\rm eff}\simeq|\langle v^s_j\rangle-\langle v^{s'}_j\rangle|/\lambda^{ss'}$, thus Eq. (15) becomes

$$(A^s_j)_{\rm coll}-(A^{s'}_j)_{\rm coll}=-(-1)^s 2\rho^{s'}\frac{(\langle v^s_j\rangle-\langle v^{s'}_j\rangle)^2}{\lambda^{ss'}}. \qquad (17)$$

In general, Eq. (15) has the form

$$(A_j^s)_{\rm coll}-(A_j^{s'})_{\rm coll}=-(-1)^s\frac{2\rho^s\rho^{s'}}{\rho^s+\rho^{s'}}\frac{(\langle v_j^s\rangle-\langle v_j^{s'}\rangle)^2}{\lambda^{ss'}}. \quad (18)$$

Substituting Eq. (18) into Eq. (12) leads to

$$(\rho^s+\kappa^s\rho^{s'})\frac{d^s\langle v_j^s\rangle}{dt}-(\rho^{s'}+\kappa^{s'}\rho^s)\frac{d^{s'}\langle v_j^{s'}\rangle}{dt}$$

$$=(\rho^s-\rho^{s'})g_j-(-1)^s\frac{2\rho^s\rho^{s'}}{\rho^s+\rho^{s'}}\frac{(\langle v_j^s\rangle-\langle v_j^{s'}\rangle)^2}{\lambda^{ss'}}. \quad (19)$$

Applying Eq. (19) to the edges of the mixing zone, at the edge of fluid s, $\langle v_j^{s'}\rangle=0$, the edge velocity $V_j^s\equiv\langle v_j^s\rangle$. Thus the nonlinear dynamic evolution equation for the edge s is

$$(\rho^s+\kappa^s\rho^{s'})\frac{d^sV_j^s}{dt}=(\rho^s-\rho^{s'})g_j-(-1)^s\frac{2\rho^s\rho^{s'}}{\rho^s+\rho^{s'}}\frac{V_j^{s2}}{\lambda^{ss'}}. \quad (20)$$

In order to compare with experiments, we now consider one-dimensional very weakly compressible or incompressible flow. In this physical situation, $\delta_{ij}V_i^s\partial V_j^s/\partial x_i\approx 0$. In terms of Atwood number $A\equiv(-1)^s(\rho^s-\rho^{s'})/(\rho^s+\rho^{s'})$, $\rho^s=[1+(-1)^sA]/2$, Eq. (20) reduces to

$$(\rho^s+\kappa^s\rho^{s'})\frac{\partial V_j^s}{\partial t}=(\rho^s-\rho^{s'})g_j-(-1)^s[1+(-1)^sA]\frac{\rho^{s'}V_j^{s2}}{\lambda^{ss'}}. \quad (21)$$

Notice that Eq. (21) is just the phenomenological buoyancy-drag equation for incompressible flow studied by a number of authors [11,12,14], which has the form

$$(\rho_i+k_i\rho_{i'})\frac{\partial V_i}{\partial t}=(\rho_i-\rho_{i'})g(t)-(-1)^i\frac{C_i\rho_{i'}V_i^2}{|Z_i|}, \quad i=1,2, \quad (22)$$

where $V_i\equiv dZ_i/dt$ is the velocity of the edge i of the mixing zone. Z_i is the position of edge i, and it equals the half width of the mixing layer, $Z_i=L/2$, in considering the integrated effects of reflected shocks. C_i is the drag coefficient of fluid i due to the existence of fluid i'.

Here it is worthwhile to point out that Eq. (21) is obtained from both the momentum equation and the boundary condition (13) in the incompressible limit. Sometimes it is used as an alternative boundary condition to close the model equations in incompressible fluids [13]. However, such a closure seems unnatural because the edge velocity is usually given by the flow velocity at the edge after the velocity field of the flow is fully solved together with the boundary conditions, but not vice versa.

IV. PARAMETER $\alpha^{ss'}$ AND DRAG COEFFICIENT

In terms of the width of the mixing layer L, Eqs. (21) and (22) are written as

$$(\rho^s+\kappa^s\rho^{s'})\frac{\partial V_j^s}{\partial t}=(\rho^s-\rho^{s'})g_j-(-1)^s[1+(-1)^sA]\frac{\rho^{s'}V_j^{s2}}{\lambda_c+\alpha^{ss'}L} \quad (23)$$

and

$$(\rho_i+k_i\rho_{i'})\frac{\partial V_i}{\partial t}=(\rho_i-\rho_{i'})g(t)-(-1)^i\frac{2C_i\rho_{i'}V_i^2}{L}. \quad (24)$$

Comparing these two equations, we obtain

$$\alpha^{ss'}=\frac{1+(-1)^sA}{2C_s}-\frac{\lambda_c}{L}, \quad s=1,2. \quad (25)$$

Usually, the normal plasma mean free path (λ_c) is many orders of magnitude smaller than the width of the mixing layer so that λ_c/L is negligible. Thus Eq. (25) gives

$$\alpha^{ss'}\approx\frac{1+(-1)^sA}{2C_s}. \quad (26)$$

This equation reveals the physics of the model parameter $\alpha^{ss'}$ in the multifluid interpenetration mix model [1]. It shows that the model parameter $\alpha^{ss'}$, which describes the turbulence enhancement of the mean free path of particle s, in fact, is the inverse of the drag coefficient between fluids s and s' which solely depends on the shape and velocity of the bubbles or spikes of fluid s. This relationship provides a physical connection between the microscopic quantities, such as the collision frequency and mean free path ($\lambda^{ss'}$) of particles s, and the macroscopic physical quantities, e.g., the fluid drag coefficient C_s. In engineering systems, generally, the faster the flow, the smaller the drag, and the bigger the $\alpha^{ss'}$.

Let $s=1$ be the light fluid (light particles) and $s=2$ represent the heavy fluid (heavy particles), then the turbulence enhancement to the mean free path of light particles and heavy particles (relative to L), correspondingly, becomes

$$\alpha^{12}=\frac{1-A}{2C_1} \quad \text{and} \quad \alpha^{21}=\frac{1+A}{2C_2}. \quad (27)$$

These expressions show that the augmentation of the mean free path of the particles due to turbulence varies with the Atwood number. It increases with A for heavy particles, but decreases with$A$ for light particles. Equal enhancement only occurs at $A=0$. Therefore, for fluid systems with large A, the dynamical evolution of the mixing zone is mainly dominated by the dynamics of the heavy particles. For this reason, we focus attention on the estimation of the enhancement for heavy particles, i.e., α^{21}.

The drag coefficient has been evaluated for isolated bubbles [26] but not for interpenetrating fluids. For interpenetrating fluids, the inferred drag coefficient from the Layzer [27] single-mode equation $A=1$ limit, Besnard-Harlow-

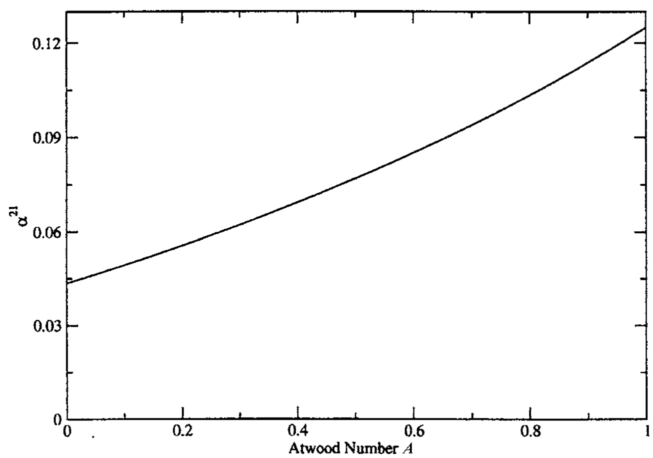

FIG. 1. The predicted α^{21} from Eq. (29).

Rauenzahn (BHR) model [9] and the hybrid mix model [18] are, respectively, $C_2 \sim 2\pi$ and $C_2 \sim 5.3$, where C_2 is assumed to be independent of the Atwood number. With these values, the possible range for parameter α^{21} is $\sim 0.08-0.18$. An A dependent drag coefficient has also been proposed by a number of authors [8,14,28]. The best determination of C_2 was given by Youngs [8]

$$C_2 \simeq 11.5 - 3.5A \tag{28}$$

in obtaining a good agreement with the observed Rocket-Rig experimental data [4,8]. Substituting (28) into Eq. (27) leads to

$$\alpha^{21} \simeq \frac{1+A}{2(11.5-3.5A)}. \tag{29}$$

By this formula, we can calculate the theoretical range of α^{21} for all Atwood numbers. The calculated results are plotted in Fig. 1, from which we see that the physical values of the mean free path enhancement for heavy particles due to turbulence, i.e., the model parameter α^{21} in the multifluid interpenetration mix model, inferred from the Rocket-Rig experiments, is in the range of

$$\alpha^{21} \sim 0.043 - 0.125 \tag{30}$$

for all As. These numbers are in good agreement with the values $\alpha^{21} \sim 0.07-0.125$ observed in various turbulent systems [29] and ICF capsule calculations $\alpha^{21} \sim 0.05-0.1$ [21,22,30].

It is worthwhile to point out that the values given in (30) are for highly compressed fluids. If the fluids are less compressed, the values of α^{21} will be slightly smaller than those in (30). The reason for this is that fluid compressibility acts like a drag [24]. Increasing compressibility increases drag, therefore, it decreases $\alpha^{ss'}$. Also if the ratio λ_c/L in a flow system is of the order $O(10^{-2}-10^{-3})$, then the values of α^{21} will, too, decrease slightly. Thus, (30) actually imposes an upper limit for the model parameter $\alpha^{ss'}$ in the multifluid interpenetration mix model. Finally, we would like to add that in practice, the mass density and drag of the compressed fluids vary with experiments so that a slightly varied value of α^{21} should be expected for similar experimental situations. This has been seen in ICF capsule implosions.

V. CONCLUSION

In this paper, we have derived the phenomenological buoyancy-drag equation from the multifluid interpenetration mix model equations and established a physical connection between the microscopic model parameters, such as the collision frequencies, mean free path, etc., and the macroscopic physical quantities, e.g., the drag coefficient. This derivation reveals the physical meaning of the only free parameter $\alpha^{ss'}$ in the multifluid interpenetration mix model, $\alpha^{ss'} \simeq [1+(-1)^s A]/(2C_s)$. As shown in this paper, this parameter depends on the density difference and the drag coefficient between fluids and can be evaluated indirectly from RT and RM experiments. For a best fit to the Rocket-Rig experimental data, the predicted values for parameter α^{21} of heavy particles is in the range of $\alpha^{21} \sim 0.043-0.125$ depending on the Atwood number. These values are in good agreement with the fitted α^{21} in ICF implosions.

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